Online Appendix
Navigating Optimal Inflation Targeting in South Africa: A Regime-Switching DSGE Approach

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This appendix provides supplementary materials not included in the article. Section A describes the data. The model equations and the first order conditions are presented under section B. Section C briefly explains the regime switching framework. In section D, a simplified open DSGE model is derived to stress the importance of commodity shocks in explaining business cycle fluctuations in South Africa. Sections E plots the IRFs of the estimated model with confidence bands. Section F compares the IRFs of the regime switching with those of a constant paramters model. Section G presents the variance decomposition of selected variables.

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A Data and Transformations

The model is estimated using South African quarterly data from 2000Q1 to 2019Q2. There are 12 observables. The exchange rate depreciation, the commodity-based inflation, GDP growth, the short-term interest rate, the current account growth rate, the CPI-based inflation, household consumption expenditure, the real wage, the government expenditure and investment. For the foreign block represented by the US, we have used foreign inflation, output, and the shadow federal funds rate of Wu and Xia (2016). Below are the descriptions of the series used.

- Exchange rate depreciation: log(1/REER)/(1/REER(-1))
- Commodity-based inflation: log(ComP/ComP(-1));
- GDP growth: log(GDP/GDP(-1));
- Consumption growth: log(CONS/CONS(-1));
- Wage inflation: log(W/W(-1));
- Short-term interest rate: log(1+R/100)ˆ1/4;
- Current account growth: log(CA/CA(-1));
- CPI-based inflation: log(CPI/CPI(-1));
- Investment growth: log(INV/INV(-1));
- Foreign GDP growth: log(US GDP/US GDP(-1));
- Foreign Short-term interest rate: log(1+US FFR/100)ˆ1/4;
- Foreign CPI-based inflation: log(US CPI/US CPI(-1));

All series, but the commodity-based inflation, were detrended using their respective sample averages. The commodity-based inflation was detrended using the HP filter.

B Model Equations and First-Order Conditions (Log-Linearized form)

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This appendix presents all 71 log-linearized equations of the DSGE model for South Africa, directly translated from the RISE implementation, extending Medina and Soto (2007). Variables are log-deviations from steady state (e.g., $\hat{x}_t = \log x_t - \log x$), and parameters are defined in a calibration table in the main text. Equations are organized by sector, with AR(1) processes (except commodity production) in a separate subsection, followed by steady-state relationships. The 72 endogenous variables match the numbered equations, ensuring model solvability.

B.1 Households

$$\begin{aligned} &-\hat{c}_t + \lambda \frac{W_r}{Y_L C_Y}(\hat{w}_t + \hat{l}_t) - \lambda \frac{\tau_Y}{C_Y}(\hat{\tau}_t + \hat{y}_t + \hat{p}_{Y,t}) - (1-\lambda)\frac{1-h}{1+h}\sigma_C(\hat{i}_t - \hat{\pi}_{C,t+1}) \\ &\quad + \frac{1}{1+h}\hat{c}_{t+1} - \lambda \frac{1}{1+h}\frac{W_r}{Y_L C_Y}(\hat{w}_{t+1} + \hat{l}_{t+1}) + \lambda \frac{1}{1+h}\frac{\tau_Y}{C_Y}(\hat{\tau}_{t+1} + \hat{y}_{t+1} + \hat{p}_{Y,t+1}) \\ &\quad + \frac{h}{1+h}\hat{c}_{t-1} - \lambda \frac{h}{1+h}\frac{W_r}{Y_L C_Y}(\hat{w}_{t-1} + \hat{l}_{t-1}) + \lambda \frac{h}{1+h}\frac{\tau_Y}{C_Y}(\hat{\tau}_{t-1} + \hat{y}_{t-1} + \hat{p}_{Y,t-1}) \\ &\quad + (1-\lambda)\frac{1-h}{1+h}(1-\rho_{sh_c})\hat{sh}_{c,t} - (1-\lambda)\frac{h-\rho_T}{1+h}\hat{s}_t = 0, \end{aligned} \tag{B-1}$$

where $\hat{c}_t$ is consumption, λ is the non-Ricardian share, W_r, Y_L, C_Y, τ_Y are steady-state ratios, $\hat{w}_t$ is the real wage, $\hat{l}_t$ is labor, $\hat{\tau}_t$ is taxes, $\hat{y}_t$ is GDP, $\hat{p}_{Y,t}$ is the GDP deflator, h is habit persistence, σ_C is the intertemporal elasticity, $\hat{i}_t$ is the interest rate, $\hat{\pi}_{C,t}$ is CPI inflation, ρ_{sh_c}, $\hat{sh}_{c,t}$ are preference shock parameters, ρ_T, $\hat{s}_t$ are permanent productivity shock parameters.

$$-\hat{i}_t + \hat{i}_{F,t} + \varrho \hat{b}_{F,t} + (1-\varrho_{de})\hat{\mathcal{E}}_{t+1} - \varrho_{de}\hat{\mathcal{E}}_t = 0, \tag{B-2}$$

where $\hat{i}_{F,t}$ is the foreign interest rate, ϱ is the risk premium elasticity, $\hat{b}_{F,t}$ is foreign bonds, ϱ_{de} is the exchange rate weight, and $\hat{\mathcal{E}}_t$ is nominal exchange rate depreciation.

$$
\begin{aligned}
&- (1 + \beta\phi_L^2 + \sigma_L\epsilon_L\phi_L(1+\beta))\hat{w}_t + (1-\beta\phi_L)(1-\phi_L)\left(\sigma_L\hat{l}_t + \frac{1}{1-h}\hat{c}_t - \frac{h}{1-h}\hat{c}_{t-1} + \hat{sh}_{w,t}\right) \\
&+ (1+\sigma_L\epsilon_L)\phi_L\hat{w}_{t-1} + (1+\sigma_L\epsilon_L)\beta\phi_L\hat{w}_{t+1} - (1+\sigma_L\epsilon_L)\phi_L(1+\beta\xi_L)\hat{\pi}_{C,t} \\
&+ (1+\sigma_L\epsilon_L)\phi_L\xi_L\hat{\pi}_{C,t-1} + (1+\sigma_L\epsilon_L)\beta\phi_L\hat{\pi}_{C,t+1} = 0,
\end{aligned}
\tag{B-3}
$$

where σ_L is the inverse Frisch elasticity, ϵ_L is labor elasticity, ϕ_L is the Calvo probability, ξ_L is wage indexation, β is the discount factor, and $\hat{sh}_{w,t}$ is the labor supply shock.

$$
-\hat{c}_{H,t} + \hat{c}_t - [\eta_C(1-\gamma_C) + \omega_C\gamma_C]\hat{p}_{H,t} + (\eta_C - \omega_C)(1-\gamma_C)\hat{p}_{F,t} = 0, \tag{B-4}
$$

where $\hat{c}_{H,t}$ is domestic goods consumption, η_C is the substitution elasticity, γ_C is the home goods share, ω_C is the oil substitution elasticity, $\hat{p}_{H,t}$ is the home goods price, and $\hat{p}_{F,t}$ is the foreign goods price.

$$
-\hat{c}_{F,t} + \hat{c}_t - [\eta_C\gamma_C + \omega_C(1-\gamma_C)]\hat{p}_{F,t} + (\eta_C - \omega_C)\gamma_C\hat{p}_{H,t} = 0, \tag{B-5}
$$

where $\hat{c}_{F,t}$ is imported goods consumption.

$$
-\hat{o}_{C,t} + \hat{c}_t - \omega_C(\hat{r}_t + \hat{p}_{O,t}) = 0, \tag{B-6}
$$

where $\hat{o}_{C,t}$ is oil consumption, $\hat{r}_t$ is the real exchange rate, and $\hat{p}_{O,t}$ is the oil price.

$$
\alpha_C(\hat{r}_t + \hat{p}_{O,t}) + (1-\alpha_C)\gamma_C\hat{p}_{H,t} + (1-\alpha_C)(1-\gamma_C)\hat{p}_{F,t} = 0, \tag{B-7}
$$

where α_C is the core consumption share.

$$
-\hat{\pi}_{Z,t} + \gamma_C\hat{\pi}_{H,t} + (1-\gamma_C)\hat{\pi}_{F,t} = 0, \tag{B-8}
$$

where $\hat{\pi}_{Z,t}$ is core inflation, $\hat{\pi}_{H,t}$ is domestic goods inflation, and $\hat{\pi}_{F,t}$ is imported goods inflation.

$$
-\Delta\hat{c}_t + \hat{c}_t - \hat{c}_{t-1} + \hat{s}_t = 0, \tag{B-9}
$$

where $\Delta\hat{c}_t$ is consumption growth.

$$-\Delta\hat{w}_t + \hat{w}_t - \hat{w}_{t-1} + \hat{s}_t = 0, \tag{B-10}$$

where $\Delta\hat{w}_t$ is real wage growth.

B.2 Firms

$$\begin{aligned}
&-\hat{y}_{H,t} + \hat{a}_{H,t} + \gamma_H^{1/\omega_H}\left(\frac{A_H O_{H,L}}{Y_{H,L}}\right)^{\frac{\omega_H-1}{\omega_H}}\hat{o}_{H,t} \\
&\quad + (1-\gamma_H)^{1/\omega_H}\left(\frac{A_H G_{H,L}}{Y_{H,L}}\right)^{\frac{\omega_H-1}{\omega_H}}\eta_H^{1/\theta_H}\left(\frac{1}{G_{H,L}}\right)^{\frac{\theta_H-1}{\theta_H}}\hat{l}_t \\
&\quad + (1-\gamma_H)^{1/\omega_H}\left(\frac{A_H G_{H,L}}{Y_{H,L}}\right)^{\frac{\omega_H-1}{\omega_H}}(1-\eta_H)^{1/\theta_H}\left(\frac{1}{G_{H,K}}\right)^{\frac{\theta_H-1}{\theta_H}}(\hat{k}_{t-1} - \hat{s}_t + \hat{u}_t) = 0,
\end{aligned} \tag{B-11}$$

where $\hat{y}_{H,t}$ is home goods output, $\hat{a}_{H,t}$ is transitory productivity, γ_H, ω_H, η_H, θ_H are production parameters, A_H, $O_{H,L}$, $Y_{H,L}$, $G_{H,L}$, $G_{H,K}$ are steady-state terms, $\hat{o}_{H,t}$ is oil input, $\hat{k}_{t-1}$ is capital, and $\hat{u}_t$ is capital utilization.

$$\hat{y}_{S,t} = \rho_{y_S}\hat{y}_{S,t-1} + \sigma_{y_S}\varepsilon_{y_S,t}, \tag{B-12}$$

where $\hat{y}_{S,t}$ is commodity output, ρ_{y_S} is persistence, σ_{y_S} is the shock scale, and $\varepsilon_{y_S,t}$ is the commodity shock.

$$-\phi_H\hat{\pi}_{H,t} + \phi_H\frac{\beta}{1+\beta\xi_H}\hat{\pi}_{H,t+1} + \phi_H\frac{\xi_H}{1+\beta\xi_H}\hat{\pi}_{H,t-1} + \frac{(1-\beta\phi_H)(1-\phi_H)}{1+\beta\xi_H}(\hat{mc}_{H,t} - \hat{p}_{H,t}) = 0, \tag{B-13}$$

where ϕ_H is the Calvo probability, ξ_H is indexation, $\hat{\pi}_{H,t}$ is domestic goods inflation, $\hat{mc}_{H,t}$ is marginal cost, and $\hat{p}_{H,t}$ is the real price.

$$-\phi_F\hat{\pi}_{H,F,t} + \phi_F\frac{\beta}{1+\beta\xi_F}\hat{\pi}_{H,F,t+1} + \phi_F\frac{\xi_F}{1+\beta\xi_F}\hat{\pi}_{H,F,t-1} + \frac{(1-\beta\phi_F)(1-\phi_F)}{1+\beta\xi_F}(\hat{mc}_{H,t} - \hat{r}_t - \hat{p}_{H,F,t}) = 0, \tag{B-14}$$

where ϕ_F is the Calvo probability, ξ_F is indexation, $\hat{\pi}_{H,F,t}$ is export goods inflation, and $\hat{p}_{H,F,t}$ is the export price.

$$
\begin{aligned}
&-\phi_F \hat{\pi}_{F,t} + \phi_F \frac{\beta}{1+\beta\xi_F}\hat{\pi}_{F,t+1} + \phi_F \frac{\xi_F}{1+\beta\xi_F}\hat{\pi}_{F,t-1} \\
&\quad + \frac{(1-\beta\phi_F)(1-\phi_F)}{1+\beta\xi_F}(\hat{r}_t + \hat{p}_{F,F,t} - \hat{p}_{F,t}) = 0,
\end{aligned} \tag{B-15}
$$

where $\hat{\pi}_{F,t}$ is import goods inflation, and $\hat{p}_{F,F,t}$ is the foreign import price shock.

$$
\begin{aligned}
&-\hat{mc}_{H,t} + \frac{Z_r K_{H,L}}{Y_{H,L}}\frac{\epsilon_H}{\epsilon_H - 1}(\hat{z}_t + \hat{k}_{t-1} - \hat{s}_t + \hat{u}_t) + \frac{W_r}{Y_{H,L}}\frac{\epsilon_H}{\epsilon_H - 1}(\hat{w}_t + \hat{l}_t) \\
&\quad + \frac{P_{r,O} O_{H,L}}{Y_{H,L}}\frac{\epsilon_H}{\epsilon_H - 1}(\hat{r}_t + \hat{p}_{O,t} + \hat{o}_{H,t}) - \hat{y}_{H,t} = 0,
\end{aligned} \tag{B-16}
$$

where Z_r, $K_{H,L}$, W_r, $P_{r,O}$, $O_{H,L}$, $Y_{H,L}$ are steady-state terms, ϵ_H is the demand elasticity, and $\hat{z}_t$ is the capital rental rate.

$$
\begin{aligned}
&\frac{1}{\omega_H}\hat{o}_{H,t} - \left[\left(\frac{1}{\omega_H} + \frac{1}{\theta_H}\right)\eta_H^{1/\theta_H} G_{H,L}^{-\frac{\theta_H - 1}{\theta_H}} - \frac{1}{\theta_H}\right]\hat{l}_t \\
&\quad - \left[\left(\frac{1}{\omega_H} + \frac{1}{\theta_H}\right)(1-\eta_H)^{1/\theta_H} G_{H,K}^{-\frac{\theta_H - 1}{\theta_H}}\right](\hat{k}_{t-1} + \hat{u}_t - \hat{s}_t) \\
&\quad + \hat{r}_t + \hat{p}_{O,t} - \hat{w}_t = 0,
\end{aligned} \tag{B-17}
$$

where $G_{H,L}$, $G_{H,K}$ are steady-state terms.

$$
-\hat{c}_{H,F,t} + \hat{y}_{F,t} - \eta_F \hat{p}_{H,F,t} = 0, \tag{B-18}
$$

where $\hat{c}_{H,F,t}$ is foreign demand, $\hat{y}_{F,t}$ is foreign output, and η_F is the foreign demand elasticity.

$$
\begin{aligned}
&-\hat{k}_t + \frac{1-\delta_I}{(1+n)(1+g_y)}\hat{k}_{t-1} + \left(1 - \frac{1-\delta_I}{(1+n)(1+g_y)}\right)(\hat{inv}_t + \hat{sh}_{i,t}) \\
&\quad - \frac{Z_r}{P_{r,I}(1+n)(1+g_y)}\hat{u}_t - \frac{1-\delta_I}{(1+n)(1+g_y)}\hat{s}_t = 0,
\end{aligned} \tag{B-19}
$$

where $\hat{k}_t$ is capital, δ_I is depreciation, n is population growth, g_y is steady-state growth, $\hat{inv}_t$ is investment, $\hat{sh}_{i,t}$ is the investment shock, Z_r, $P_{r,I}$ are steady-state terms, and $\hat{u}_t$ is capital utilization.

$$
-\hat{inv}_{H,t} + \hat{inv}_t - \theta_I(\hat{p}_{H,t} - \hat{p}_{I,t}) = 0, \tag{B-20}
$$

where $\hat{inv}_{H,t}$ is domestic goods investment, θ_I is the investment elasticity, and $\hat{p}_{I,t}$ is the investment price index.

$$-\hat{inv}_{F,t} + \hat{inv}_t - \theta_I(\hat{p}_{F,t} - \hat{p}_{I,t}) = 0, \tag{B-21}$$

where $\hat{inv}_{F,t}$ is foreign goods investment.

$$-\hat{p}_{I,t} + \gamma_I \hat{p}_{H,t} + (1-\gamma_I)\hat{p}_{F,t} = 0, \tag{B-22}$$

where γ_I is the domestic goods share in investment.

$$\begin{aligned} -\hat{p}_{I,t} + \hat{q}_t + \hat{sh}_{i,t} - \left(1 + \frac{1}{1+r}\right)(1+g_y)^2 S_I \hat{inv}_t + (1+g_y)^2 S_I \hat{inv}_{t-1} \\ + \frac{(1+g_y)^2 S_I}{1+r}\hat{inv}_{t+1} - (1+g_y)^2 S_I \left(1 - \frac{\rho_T(1+g_y)}{1+r}\right)\hat{s}_t = 0, \end{aligned} \tag{B-23}$$

where $\hat{q}_t$ is the price of capital, S_I is the adjustment cost parameter, and r is the steady-state real interest rate.

$$-\hat{q}_t + \hat{\pi}_{C,t+1} - \hat{i}_t + \frac{Z_r}{P_{r,I}(1+r)(1+\text{premio}_s)}\hat{z}_{t+1} + \frac{1-\delta_I}{(1+r)(1+\text{premio}_s)}\hat{q}_{t+1} = 0, \tag{B-24}$$

where premio_s is the risk premium spread.

$$-\hat{z}_t + \hat{q}_t + \sigma_I \hat{u}_t = 0, \tag{B-25}$$

where σ_I is the utilization scaling parameter.

$$-\frac{1}{\theta_H}(\hat{k}_{t-1} + \hat{u}_t - \hat{s}_t - \hat{l}_t) + \hat{w}_t - \hat{z}_t = 0, \tag{B-26}$$

where θ_H is the CES production parameter.

$$-\hat{x}_t + \frac{Y_s}{X}\hat{y}_{S,t} + \left(1 - \frac{Y_s}{X}\right)\hat{c}_{H,F,t} = 0, \tag{B-27}$$

where $\hat{x}_t$ is total exports, Y_s/X is the commodity export share, and X is steady-state exports.

$$-\hat{p}_{X,t} + \frac{Y_s}{X}\hat{p}_{S,t} + \left(1 - \frac{Y_s}{X}\right)(\hat{p}_{H,F,t} + \hat{r}_t) = 0, \tag{B-28}$$

where $\hat{p}_{X,t}$ is the export price deflator, and $\hat{p}_{S,t}$ is the commodity price.

$$-\hat{m}_t + \frac{M_o}{M}\hat{m}_{o,t} + \left(1 - \frac{M_o}{M}\right)\frac{C_Y(1-\alpha_C)(1-\gamma_C)}{M_F Y}\hat{c}_{F,t} + \left(1 - \frac{M_o}{M}\right)\frac{I_Y(1-\gamma_I)}{M_F Y}\hat{inv}_{F,t} = 0, \tag{B-29}$$

where $\hat{m}_t$ is total imports, M_o/M is the oil import share, C_Y, I_Y, $M_F Y$ are steady-state ratios, and $\hat{m}_{o,t}$ is oil imports.

$$-\hat{m}_{o,t} + \frac{\alpha_C C_Y}{M_Y M_o}\hat{o}_{C,t} + \left(1 - \frac{\alpha_C C_Y}{M_Y M_o}\right)\hat{o}_{H,t} = 0, \tag{B-30}$$

where M_Y, M_o are steady-state terms.

$$-\hat{p}_{M,t} + \frac{M_o}{M}\hat{p}_{O,t} + \left(1 - \frac{M_o}{M}\right)\hat{p}_{F,F,t} + \hat{r}_t = 0, \tag{B-31}$$

where $\hat{p}_{M,t}$ is the import price deflator.

$$-\hat{p}_{S,t} + \hat{p}^*_{S,t} + \hat{r}_t = 0, \tag{B-32}$$

where $\hat{p}^*_{S,t}$ is the international commodity price.

$$-\hat{p}_{S,a,t} + \hat{r}_t + \frac{p_{S,F,\mathrm{ref}}(1 - \rho_{p^*_S}^{n_{p^*_S}})}{(1-\rho_{p^*_S})n_{p^*_S}}\hat{p}^*_{S,t} = 0, \tag{B-33}$$

where $\hat{p}_{S,a,t}$ is the adjusted commodity price, $p_{S,F,\mathrm{ref}}$, $n_{p^*_S}$ are steady-state terms, and $\rho_{p^*_S}$ is persistence.

$$-\hat{\pi}_{H,t} + \hat{p}_{H,t} - \hat{p}_{H,t-1} + \hat{\pi}_{C,t} = 0, \tag{B-34}$$

$$-\hat{\pi}_{H,F,t} + \hat{p}_{H,F,t} - \hat{p}_{H,F,t-1} + \hat{\pi}_{F,F,t} = 0, \tag{B-35}$$

where $\hat{\pi}_{F,F,t}$ is foreign inflation.

$$-\hat{\pi}_{F,t} + \hat{p}_{F,t} - \hat{p}_{F,t-1} + \hat{\pi}_{C,t} = 0, \tag{B-36}$$

$$-\hat{\mathcal{E}}_t + \hat{r}_t - \hat{r}_{t-1} + \hat{\pi}_{C,t} - \hat{\pi}_{F,F,t} = 0, \tag{B-37}$$

$$-\Delta \hat{inv}_t + \hat{inv}_t - \hat{inv}_{t-1} + \hat{s}_t = 0, \tag{B-38}$$

where $\Delta \hat{inv}_t$ is investment growth.

$$-\hat{k}_{lag,t} + \hat{k}_{t-1} = 0, \tag{B-39}$$

where $\hat{k}_{lag,t}$ is lagged capital.

$$\begin{aligned} -\hat{y}_{H,t} + \frac{\gamma_C C_Y}{Yh_Y}\hat{c}_{H,t} + \frac{G_h}{Yh_Y}(\hat{g}_t - \hat{p}_{H,t} + \hat{p}_{Y,t} + \hat{y}_t) \\ + \frac{\gamma_I I_Y}{Yh_Y}\hat{inv}_{H,t} + \frac{YhF_Y}{Yh_Y}\hat{c}_{H,F,t} = 0, \end{aligned} \tag{B-40}$$

where Yh_Y, YhF_Y are steady-state ratios.

$$-\hat{y}_t + C_Y\hat{c}_t + G_h(\hat{g}_t - \hat{p}_{H,t} + \hat{p}_{Y,t} + \hat{y}_t) + I_Y\hat{inv}_t + X_Y\hat{x}_t - M_Y\hat{m}_t = 0, \tag{B-41}$$

where M_Y is the imports ratio.

$$\begin{aligned} &-\frac{eBF_Y}{(1+i_F)\Theta_{ss}}(\hat{b}_{F,t} - \hat{i}_{F,ax,t}) + \frac{eBF_Y}{(1+\pi_F)(1+n)(1+g_y)}(\hat{\mathcal{E}}_t - \hat{\pi}_{C,t} + \hat{p}_{Y,t-1} + \hat{y}_{t-1} - \hat{p}_{Y,t} - \hat{y}_t + \hat{b}_{F,t-1} - \hat{s}_t) \\ &- (1-\chi)Y_s(\hat{r}_t + \hat{p}^*_{S,t} + \hat{y}_{S,t} - \hat{p}_{Y,t} - \hat{y}_t) + X_Y(\hat{p}_{X,t} + \hat{x}_t - \hat{p}_{Y,t} - \hat{y}_t) \\ &- M_Y(\hat{p}_{M,t} + \hat{m}_t - \hat{p}_{Y,t} - \hat{y}_t) = 0, \end{aligned} \tag{B-42}$$

where eBF_Y is the bond-to-GDP ratio.

$$\begin{aligned} -\hat{ca}_y &+ (1-\chi)Y_s(\hat{r}_t + \hat{p}^*_{S,t} + \hat{y}_{S,t}) + X_Y(\hat{p}_{X,t} + \hat{x}_t - \hat{p}_{Y,t} - \hat{y}_t) \\ &- M_Y(\hat{p}_{M,t} + \hat{m}_t - \hat{p}_{Y,t} - \hat{y}_t) + \frac{eBF_Y}{(1+\pi_F)(1+n)(1+g_y)(1+i_F)\Theta_{ss}}\hat{i}_{F,ax,t-1} \\ &+ \frac{eBF_Y[(1+i_F)\Theta_{ss} - 1]}{(1+\pi_F)(1+n)(1+g_y)(1+i_F)\Theta_{ss}}(\hat{b}_{F,t-1} + \hat{\mathcal{E}}_t - \hat{\pi}_{C,t} + \hat{p}_{Y,t-1} + \hat{y}_{t-1} - \hat{p}_{Y,t} - \hat{y}_t - \hat{s}_t) = 0, \end{aligned} \tag{B-43}$$

where $\hat{ca}_y$ is the current account-to-GDP ratio.

$$-\hat{i}_{F,ax,t} + \hat{i}_{F,t} + \varrho \hat{b}_{F,t} - \varrho_{de}(\hat{\mathcal{E}}_{t+1} + \hat{\mathcal{E}}_t) = 0, \tag{B-44}$$

where $\hat{i}_{F,ax,t}$ is the auxiliary foreign interest rate.

$$-\hat{p}_{Y,t} + G_h \hat{p}_{H,t} + I_Y \hat{p}_{I,t} + X_Y \hat{p}_{X,t} - M_Y \hat{p}_{M,t} = 0, \tag{B-45}$$

$$-\hat{r}_t + \hat{i}_t - \hat{\pi}_{Z,t} = 0, \tag{B-46}$$

where $\hat{r}_t$ is the real interest rate.

$$-\Delta \hat{y}_t + \hat{y}_t - \hat{y}_{t-1} + \hat{s}_t = 0, \tag{B-47}$$

where $\Delta \hat{y}_t$ is output growth.

$$-\hat{y}_{F,a,t} + \hat{y}_{F,t} = 0, \tag{B-48}$$

where $\hat{y}_{F,a,t}$ is auxiliary foreign output.

$$-\Delta \hat{y}_{F,t} + \hat{y}_{F,a,t} - \hat{y}_{F,a,t-1} + \hat{s}_t = 0, \tag{B-49}$$

where $\Delta \hat{y}_{F,t}$ is foreign output growth.

$$-\hat{y}_{S,a,t} + \hat{y}_{S,t} = 0, \tag{B-50}$$

where $\hat{y}_{S,a,t}$ is auxiliary commodity production.

$$-\Delta \hat{y}_{S,t} + \hat{y}_{S,a,t} - \hat{y}_{S,a,t-1} + \hat{s}_t = 0, \tag{B-51}$$

where $\Delta \hat{y}_{S,t}$ is commodity production growth.

$$-\hat{R}_{k,t} + \frac{Z_r}{P_{r,I}(1+r)(1+\text{premio}_s)} \hat{z}_t + \frac{1-\delta_I}{(1+r)(1+\text{premio}_s)} \hat{q}_t - \hat{q}_{t-1} = 0, \tag{B-52}$$

where $\hat{R}_{k,t}$ is the return on capital.

B.3 Government

$$\begin{aligned}&-G_h\hat{g}_t+\tau_Y(\hat{\tau}_t-\hat{y}_t)+\chi Y_s(\hat{p}_{S,t}+\hat{y}_{S,t}-\hat{p}_{Y,t}-\hat{y}_t)\\&\quad+\left(1-\frac{1}{\Theta(1+i_F)}\right)\frac{B_G}{(1+\pi_F)(1+g_y)(1+n)}(\hat{\mathcal{E}}_t-\hat{\pi}_{C,t}+\hat{b}_{G,t-1}-\hat{p}_{Y,t}+\hat{p}_{Y,t-1}-\hat{y}_t+\hat{y}_{t-1}-\hat{s}_t)\\&\quad+\frac{1}{\Theta(1+i_F)}\frac{B_G}{(1+n)(1+g_y)(1+n)}\hat{i}_{F,ax,t-1}+G_h(\hat{g}_{ex,t}+\hat{p}_{H,t}-\hat{p}_{Y,t}-\hat{y}_t)=0,\end{aligned}\tag{B-53}$$

where G_h, τ_Y, Y_s, B_G, π_F, i_F, n are steady-state terms, χ is the commodity revenue share, $\hat{g}_t$ is government expenditure, $\hat{b}_{G,t}$ is government foreign assets, $\hat{g}_{ex,t}$ is the expenditure shock, and $\hat{i}_{F,ax,t}$ is an auxiliary foreign interest rate.

$$\psi_\tau\hat{\tau}_t-(1-\psi_\tau)\hat{g}_t=0,\tag{B-54}$$

where ψ_τ weights the tax rule.

$$\begin{aligned}&-\frac{B_G}{(1+i_F)\Theta}(\hat{b}_{G,t}-\hat{i}_{F,ax,t})+\frac{B_G}{(1+\pi_F)(1+g_y)(1+n)}(\hat{\mathcal{E}}_t-\hat{\pi}_{C,t}+\hat{b}_{G,t-1}+\hat{p}_{Y,t-1}+\hat{y}_{t-1}-\hat{p}_{Y,t}-\hat{y}_t-\hat{s}_t)\\&+\tau_Y\hat{\tau}_t+\chi Y_s(\hat{p}^*_{S,t}+\hat{r}_t+\hat{y}_{S,t}-\hat{p}_{Y,t}-\hat{y}_t-\hat{s}_t)-G_h\hat{g}_t=0,\end{aligned}\tag{B-55}$$

B.4 Monetary Authority

$$-\hat{i}_t+\rho_r\hat{i}_{t-1}+(1-\rho_r)[\alpha_\pi(s_t)\hat{\pi}_{Z,t}+\alpha_y(s_t)\Delta\hat{y}_t]+\hat{sh}_{m,t}=0,\tag{B-56}$$

where ρ_r is smoothing, $\alpha_\pi(s_t)=\alpha_{\pi,c}+\sigma_{p^*_S,s_t}\alpha_{\pi,s}$, $\alpha_y(s_t)=\alpha_{y,c}+\sigma_{p^*_S,s_t}\alpha_{y,s}$ are volatility-dependent coefficients, $\hat{\pi}_{Z,t}$ is core inflation, $\Delta\hat{y}_t$ is output growth, and $\hat{sh}_{m,t}$ is the monetary policy shock.

$$\Delta\hat{i}_t-\hat{i}_t+\hat{i}_{t-1}=0,\tag{B-57}$$

where $\Delta\hat{i}_t$ is the interest rate change.

$$\Pi_t - \hat{\pi}_{Z,t} - \pi_C = 0, \tag{B-58}$$

where Π_t is the inflation rate, and π_C is steady-state CPI inflation.

B.5 AR(1) Processes

$$\hat{a}_{H,t} = \rho_{a_H} \hat{a}_{H,t-1} + \sigma_{a_H} \varepsilon_{a_H,t}, \tag{B-59}$$

where ρ_{a_H} is persistence, σ_{a_H} is the shock scale, and $\varepsilon_{a_H,t}$ is the shock.

$$\hat{p}^*_{S,t} = \rho_{p^*_S} \hat{p}^*_{S,t-1} + \sigma_{p^*_S,s_t} \varepsilon_{p^*_S,t}, \tag{B-60}$$

where $\rho_{p^*_S}$ is persistence, $\sigma_{p^*_S,s_t}$ is regime-dependent volatility, and $\varepsilon_{p^*_S,t}$ is the shock.

$$\hat{y}_{F,t} = \rho_{y_F} \hat{y}_{F,t-1} + \sigma_{y_F} \varepsilon_{y_F,t}, \tag{B-61}$$

where ρ_{y_F} is persistence, σ_{y_F} is the shock scale, and $\varepsilon_{y_F,t}$ is the shock.

$$\hat{i}_{F,t} = \rho_{i_F} \hat{i}_{F,t-1} + \sigma_{i_F} \varepsilon_{i_F,t}, \tag{B-62}$$

where ρ_{i_F} is persistence, σ_{i_F} is the shock scale, and $\varepsilon_{i_F,t}$ is the shock.

$$\hat{\pi}_{F,F,t} = \rho_{\pi_F} \hat{\pi}_{F,F,t-1} + \sigma_{\pi_F} \varepsilon_{\pi_F,t}, \tag{B-63}$$

where ρ_{π_F} is persistence, σ_{π_F} is the shock scale, and $\varepsilon_{\pi_F,t}$ is the shock.

$$\hat{sh}_{m,t} = \rho_{sh_m} \hat{sh}_{m,t-1} + \sigma_{sh_m} \varepsilon_{sh_m,t}, \tag{B-64}$$

where ρ_{sh_m} is persistence, σ_{sh_m} is the shock scale, and $\varepsilon_{sh_m,t}$ is the shock.

$$\hat{sh}_{w,t} = \rho_{sh_w} \hat{sh}_{w,t-1} + \sigma_{sh_w} \varepsilon_{sh_w,t}, \tag{B-65}$$

where ρ_{sh_w} is persistence, σ_{sh_w} is the shock scale, and $\varepsilon_{sh_w,t}$ is the shock.

$$\hat{sh}_{c,t} = \rho_{sh_c}\hat{sh}_{c,t-1} + \sigma_{sh_c}\varepsilon_{sh_c,t}, \tag{B-66}$$

where ρ_{sh_c} is persistence, σ_{sh_c} is the shock scale, and $\varepsilon_{sh_c,t}$ is the shock.

$$\hat{g}_{ex,t} = \rho_{gex}\hat{g}_{ex,t-1} + \sigma_{gex}\varepsilon_{gex,t}, \tag{B-67}$$

where ρ_{gex} is persistence, σ_{gex} is the shock scale, and $\varepsilon_{gex,t}$ is the shock.

$$\hat{sh}_{i,t} = \rho_{sh_i}\hat{sh}_{i,t-1} + \sigma_{sh_i}\varepsilon_{sh_i,t}, \tag{B-68}$$

where ρ_{sh_i} is persistence, σ_{sh_i} is the shock scale, and $\varepsilon_{sh_i,t}$ is the shock.

$$\hat{p}_{F,F,t} = \rho_{p_{F,F}}\hat{p}_{F,F,t-1} + \sigma_{p_{F,F}}\varepsilon_{p_{F,F},t}, \tag{B-69}$$

where $\rho_{p_{F,F}}$ is persistence, $\sigma_{p_{F,F}}$ is the shock scale, and $\varepsilon_{p_{F,F},t}$ is the shock.

$$\hat{p}_{O,t} = \rho_{p_{O,F}}\hat{p}_{O,t-1} + \sigma_{p_{O,F}}\varepsilon_{p_{O,F},t}, \tag{B-70}$$

where $\rho_{p_{O,F}}$ is persistence, $\sigma_{p_{O,F}}$ is the shock scale, and $\varepsilon_{p_{O,F},t}$ is the shock.

$$\hat{s}_t = \rho_T\hat{s}_{t-1} + \sigma_s\varepsilon_{s,t}, \tag{B-71}$$

where ρ_T is persistence, σ_s is the shock scale, and $\varepsilon_{s,t}$ is the shock.

B.6 Steady-State Relationships

$$n_F = \frac{(1+g_y)(1+n)}{1+g_{y,F}} - 1,$$

where n_F is the foreign population growth rate, g_y is domestic GDP growth, n is domestic population growth, and $g_{y,F}$ is foreign GDP growth.

$$RER = P_{r,F}\frac{\epsilon_F - 1}{\epsilon_F},$$

where RER is the steady-state real exchange rate, $P_{r,F}$ is the steady-state foreign goods price, and ϵ_F is the elasticity of demand for foreign goods.

$$P_{r,F,O} = \frac{1}{RER},$$

where $P_{r,F,O}$ is the steady-state foreign oil price.

$$P_{r,F,S} = \frac{1}{RER},$$

where $P_{r,F,S}$ is the steady-state foreign commodity price.

$$P_{r,O} = RER \cdot P_{r,F,O},$$

where $P_{r,O}$ is the steady-state domestic oil price.

$$P_{r,S} = RER \cdot P_{r,F,S},$$

where $P_{r,S}$ is the steady-state domestic commodity price.

$$\Theta_{ss} = \frac{1+r}{(1+i_F)/(1+\pi_F)},$$

where Θ_{ss} is the steady-state risk premium, r is the real interest rate, i_F is the foreign nominal interest rate, and π_F is foreign inflation.

$$Z_r = \left[(1+r)(1+\text{premio}_s) - (1-\delta_I)\right] P_{r,I},$$

where Z_r is the steady-state rental rate, premio_s is the risk premium spread, δ_I is depreciation, and $P_{r,I}$ is the investment price index.

$$eBF_Y = \frac{CCq_Y(1+i_F)\Theta_{ss}}{1 - \frac{1}{(1+n)(1+g_y)(1+\pi_F)}},$$

where eBF_Y is the bond-to-GDP ratio, CCq_Y is the risk-adjusted net foreign asset flow, and other terms are as defined.

$$Ren_Y = CCq_Y - NX_Y - \frac{eBF_Y\left(1 - \frac{1}{(1+i_F)\Theta_{ss}}\right)}{(1+n)(1+g_y)(1+\pi_F)},$$

where Ren_Y is net revenues, NX_Y is the net exports-to-GDP ratio.

$$RenNs_Y = Ren_Y + P_{r,S}Y_s(1-\chi),$$

where $RenNs_Y$ is non-commodity revenues, Y_s is commodity output, and χ is the government's commodity revenue share.

$$Kh_Lh = \left(\frac{W_r}{Z_r}\right)^{\theta_H} \frac{1-\eta_H}{\eta_H},$$

where Kh_Lh is the capital-to-labor ratio, W_r is the real wage, and η_H is the labor share.

$$Oh_Lh = \left[\frac{W_r/P_{r,O}}{\eta_H^{1/\theta_H}(Kh_Lh^{1-\eta_H})^{\frac{1/\theta_H - 1/\omega_H}{\theta_H}}}\right]^{\omega_H} \frac{\gamma_H}{1-\gamma_H},$$

where Oh_Lh is the oil-to-labor ratio, and γ_H is the oil share in production.

$$agg_Lh = \left[(1-\gamma_H)^{1/\omega_H}(Kh_Lh^{1-\eta_H})^{\frac{\omega_H-1}{\omega_H}} + \gamma_H^{1/\omega_H} Oh_Lh^{\frac{\omega_H-1}{\omega_H}}\right]^{\frac{\omega_H}{\omega_H-1}},$$

where agg_Lh is the aggregate input ratio.

$$Gh_Lh = (Kh_Lh)^{1-\eta_H},$$

where Gh_Lh is the value-added ratio.

$$Yh_Lh = \frac{(W_r + Z_rKh_Lh + P_{r,O}Oh_Lh)\epsilon_H}{\epsilon_H - 1},$$

where Yh_Lh is the output-to-labor ratio, and ϵ_H is the demand elasticity.

$$Gh_Kh = \frac{Gh_Lh}{Kh_Lh},$$

where Gh_Kh is the value-added-to-capital ratio.

$$A_h = \frac{Yh_L h}{agg_L h},$$

where A_h is productivity.

$$Id_K h = (1 + g_y)(1 + n) - (1 - \delta_I),$$

where $Id_K h$ is the investment-to-capital ratio.

$$I_Y = \frac{Id_K h K h_L h \left[1 - Y_s - (1 - NX_Y - G_h)(1 - \gamma_C)(P_{r,F} - RER)\right]}{(P_{r,H} Y h_L h - P_{r,O} O h_L h) + Id_K h K h_L h (P_{r,F} - RER)(\gamma_C - \gamma_I)},$$

where I_Y is the investment-to-GDP ratio, G_h is government spending, and $P_{r,H}$ is the home goods price.

$$C_Y = 1 - NX_Y - G_h - I_Y,$$

where C_Y is the consumption-to-GDP ratio.

$$Mf_Y = C_Y(1 - \alpha_C)(1 - \gamma_C) + I_Y(1 - \gamma_I),$$

where Mf_Y is the foreign goods imports ratio, and α_C is the core consumption share.

$$VAh_Y = 1 - Y_s - Mf_Y(P_{r,F} - RER),$$

where VAh_Y is the value-added ratio.

$$Y_L h = \frac{P_{r,H} Y h_L h - P_{r,O} O h_L h}{VAh_Y},$$

where $Y_L h$ is the GDP-to-labor ratio.

$$Yh_Y = \frac{Yh_L h}{Y_L h},$$

where Yh_Y is the home goods output ratio.

$$M_Y = RER \cdot Mf_Y + P_{r,O}\left(\frac{Oh_L h}{Y_L h} + \alpha_C C_Y\right),$$

where M_Y is the total imports ratio.

$$X_Y = NX_Y + M_Y,$$

where X_Y is the exports ratio.

$$YhF_Y = X_Y - Y_s,$$

where YhF_Y is the foreign demand ratio.

$$Mo_M = \frac{P_{r,O}\left(\frac{Oh_L h}{Y_L h} + \alpha_C C_Y\right)}{M_Y},$$

where Mo_M is the oil imports share.

$$Ys_X = \frac{Y_s}{X_Y},$$

where Ys_X is the commodity exports share.

$$bG_Y = \frac{(G_h - \tau_Y - \chi Y_s + BalG_Y)(1+\pi_C)(1+g_y)(1+n)}{1 - \frac{1}{(1+i_F)\Theta_{ss}}},$$

where bG_Y is the government assets ratio, τ_Y is tax revenue, and $BalG_Y$ is the government balance.

$$\alpha_\pi(s_t) = \alpha_{\pi,c} + \sigma_{p_S^*,s_t}\alpha_{\pi,s},$$

where $\alpha_\pi(s_t)$ is the inflation response, $\alpha_{\pi,c}$, $\alpha_{\pi,s}$ are parameters, and $\sigma_{p_S^*,s_t}$ is the volatility.

$$\alpha_y(s_t) = \alpha_{y,c} + \sigma_{p_S^*,s_t}\alpha_{y,s},$$

where $\alpha_y(s_t)$ is the output response, and $\alpha_{y,c}$, $\alpha_{y,s}$ are parameters.

B.7 Endogenous Variables

The model incorporates 71 endogenous variables, corresponding to the 71 numbered equations, spanning from (B-1) to (B-71):

1. $\hat{a}_{H,t}$: Transitory productivity
2. $\hat{b}_{F,t}$: Foreign bonds
3. $\hat{b}_{G,t}$: Government foreign assets
4. $\hat{c}_t$: Household consumption
5. $\hat{ca}_y$: Current account-to-GDP ratio
6. $\hat{c}_{F,t}$: Imported goods consumption
7. $\hat{c}_{H,t}$: Home goods consumption
8. $\hat{c}_{H,F,t}$: Foreign goods consumption (exports)
9. $\Delta\hat{c}_t$: Consumption growth
10. $\hat{\mathcal{E}}_t$: Exchange rate depreciation
11. $\Delta\hat{inv}_t$: Investment growth
12. $\Delta\hat{w}_t$: Real wage growth
13. $\Delta\hat{y}_t$: Output growth
14. $\Delta\hat{y}_{F,t}$: Foreign output growth
15. $\Delta\hat{y}_{S,t}$: Commodity production growth
16. $\hat{g}_t$: Government expenditure
17. $\hat{g}_{ex,t}$: Government expenditure shock
18. $\hat{i}_t$: Nominal interest rate

19. $\hat{i}_{F,t}$: Foreign interest rate

20. $\hat{i}_{F,ax,t}$: Auxiliary foreign interest rate

21. $\hat{inv}_t$: Total investment

22. $\hat{inv}_{F,t}$: Foreign goods investment

23. $\hat{inv}_{H,t}$: Domestic goods investment

24. $\hat{k}_t$: Capital

25. $\hat{k}_{lag,t}$: Lagged capital

26. $\hat{l}_t$: Labor supply

27. $\hat{m}_t$: Total imports

28. $\hat{mc}_{H,t}$: Marginal cost

29. $\hat{m}_{o,t}$: Oil imports

30. $\hat{o}_{C,t}$: Oil consumption

31. $\hat{o}_{H,t}$: Oil input in production

32. $\hat{\pi}_{C,t}$: CPI inflation

33. $\hat{\pi}_{Z,t}$: Core inflation

34. $\hat{\pi}_{F,t}$: Imported goods inflation

35. $\hat{\pi}_{F,F,t}$: Foreign inflation

36. $\hat{\pi}_{H,t}$: Domestic goods inflation

37. $\hat{\pi}_{H,F,t}$: Exported goods inflation

38. $\hat{p}_{F,t}$: Real price of foreign goods

39. $\hat{p}_{F,F,t}$: Real price of foreign imports

40. $\hat{p}_{H,t}$: Real price of home goods

41. $\hat{p}_{H,F,t}$: Real price of exported goods

42. $\hat{p}_{I,t}$: Investment price index

43. $\hat{p}_{M,t}$: Import price deflator

44. $\hat{p}_{O,t}$: Real oil price

45. $\hat{p}_{S,t}$: Real commodity price

46. $\hat{p}_{S,a,t}$: Adjusted commodity price

47. $\hat{p}^{*}_{S,t}$: Foreign commodity price

48. $\hat{p}_{X,t}$: Export price deflator

49. $\hat{p}_{Y,t}$: GDP deflator

50. $\hat{q}_t$: Price of capital

51. $\hat{r}_t$: Ex-post real interest rate

52. $\hat{r}_{er,t}$: Real exchange rate

53. $\hat{R}_{k,t}$: Return on capital

54. $\hat{sh}_{c,t}$: Preference shock

55. $\hat{sh}_{i,t}$: Investment adjustment cost shock

56. $\hat{sh}_{m,t}$: Monetary policy shock

57. $\hat{sh}_{w,t}$: Labor supply shock

58. $\hat{s}_t$: Permanent productivity shock

59. $\hat{\tau}_t$: Lump-sum taxes

60. $\hat{u}_t$: Capital utilization

61. $\hat{w}_t$: Real wage

62. $\hat{x}_t$: Total exports

63. $\hat{y}_t$: Real GDP

64. $\hat{y}_{F,t}$: Foreign output

65. $\hat{y}_{F,a,t}$: Auxiliary foreign output

66. $\hat{y}_{H,t}$: Home goods output

67. $\hat{y}_{S,t}$: Commodity production

68. $\hat{y}_{S,a,t}$: Auxiliary commodity production

69. $\hat{z}_t$: Capital rental rate

70. Π_t: Inflation rate

71. $\Delta\hat{i}_t$: Interest rate change

C Regime-Switching Model Formulation and Solution

In the RISE toolbox (Maih, 2015), a "regime" is defined as a composite of states from independent Markov processes, capturing joint dynamics of economic dimensions such as commodity price volatility and monetary policy stance. Specifically, we denote the regime as $r_t = \{s_t^V\}$[1], where $s_t^V \in \{H, L\}$ represents high (H) or low (L) volatility in commodity prices. Each state s_t^V reflects transitions within a single Markov process, governed by transition probabilities.

The regime-switching DSGE model is characterized by a system of expectational equations defining the equilibrium:

$$E_t \sum_{r_{t+1}=1}^{h} p_{r_t,r_{t+1}}(\mathcal{I}_t) f_{r_t}(x_{t+1}(r_{t+1}), x_t(r_t), x_{t-1}, \theta_{r_t}, \theta_{r_{t+1}}, \varepsilon_t) = 0, \tag{C-72}$$

[1] If we had multiple Markov processes, a regime could be e.g. $r_t = \{s_t^V, s_t^P\}$, with s_t^P representing the state of the additional Markov process.

where:

- $p_{r_t,r_{t+1}}(\mathcal{I}_t)$: Transition probability from regime r_t to r_{t+1}, conditional on information set $\mathcal{I}_t$.
- f_{r_t}: Nonlinear function specifying equilibrium conditions under regime r_t.
- $x_t(r_t)$: Vector of endogenous variables, regime-dependent.
- θ_{r_t}: Structural parameters, varying by regime.
- $\varepsilon_t \sim N(0, I)$: Vector of stochastic shocks.

This formulation captures forward-looking behavior, where agents' decisions in regime r_t depend on the distribution of future regimes r_{t+1}, reflecting regime-contingent expectations and precautionary adjustments.

We assume a minimum state variable (MSV) solution:

$$x_t = \mathcal{T}_{r_t}(x_{t-1}, \varepsilon_t), \tag{C-73}$$

where $\mathcal{T}_{r_t}$ maps state variables and shocks to endogenous variables, varying by regime. The state variables are:

$$z_t = [x'_{t-1}, \chi, \varepsilon'_t]', \tag{C-74}$$

with x_{t-1} as lagged endogenous variables, ε_t as shocks, and χ as a perturbation parameter.

Due to the intractability of an analytical solution, we apply a p-th order perturbation expansion around regime-specific steady states:

$$\mathcal{T}_{r_t}(z_t) \approx \mathcal{T}_{r_t}(\bar{z}_{r_t}) + \mathcal{T}_z^{r_t}(z_t - \bar{z}_{r_t}) + \frac{1}{2}\mathcal{T}_{zz}^{r_t}(z_t - \bar{z}_{r_t})^{\otimes 2} + \cdots + \frac{1}{p!}\mathcal{T}_{z^{(p)}}^{r_t}(z_t - \bar{z}_{r_t})^{\otimes p}, \tag{C-75}$$

where $\bar{z}_{r_t}$ is the steady state for regime r_t. This approach, following Maih and Waggoner (2018), accommodates switching parameters (e.g., commodity price volatility, Taylor rule coefficients) and is implemented in RISE.[2]

[2] Alternative methods include Foerster et al. (2016) and Barthélemy and Marx (2017), which also approximate solutions for regime-switching DSGE models.

D Optimal Monetary Policy in a Simplified DSGE Model

This appendix presents a simplified dynamic stochastic general equilibrium (DSGE) model for a small open economy with commodity price shocks, inspired by Drechsel, McLeay, and Tenreyro (2019). The model uses Rotemberg pricing, specified in nonlinear form to facilitate welfare-based analysis. It includes households, domestic goods firms, commodity sector firms, a monetary authority, and an exogenous foreign economy. Commodity goods are sold exclusively to the foreign economy, with profits rebated to households. A production subsidy ζ_t is optional for computational flexibility (e.g., $\zeta_t = \frac{1}{\epsilon-1}$ or $\zeta_t = 0$). The nonstationary model has 18 endogenous variables and 18 equations, ensuring solvability. The stationarized model eliminates non-stationary price levels, also with 18 endogenous variables and 18 equations. The balanced current account ($p_{f,t}c_{f,t} = p_{c,t}y_{c,t}$) is satisfied by the model's equilibrium conditions.

The model is elaborated below and we implement optimal commitment policy compared to a simple Taylor rule, to show that the commitment policy produces higher welfare and lower fluctuations. The model is calibrated for a small open economy with the same inflation target as we have in the main text. While the results on optimal commitment policy relates to a 1st-order perturbation, we are also able to implement the same exercise up to a 5th-order perturbation.

The exercise captures dynamics critical for commodity exporting economies and highlights how commodity price volatility affects π_t and $y_{h,t}$, providing theoretical insights that inform the SARB model's ad-hoc loss function (Section 5) that we implement in this paper.

D.1 Nonstationary Model

D.1.1 Households

Problem Solved: Households maximize expected lifetime utility:

$$\max_{C_t,N_t,D_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{N_t^{1+\phi}}{1+\phi} \right),$$

subject to the budget constraint:

$$P_t C_t + \frac{D_t}{1+I_t} = W_t N_t + D_{t-1} + \Psi_t, \tag{1}$$

where C_t is consumption, N_t is labor, $\beta \in (0,1)$ is the discount factor, $\phi > 0$ is the inverse Frisch elasticity, P_t is the CPI, I_t is the nominal interest rate, W_t is the wage, D_t is bond holdings, and Ψ_t is firm profits. Households consume domestic ($C_{H,t}$) and foreign goods ($C_{F,t}$) at prices $P_{H,t}$ and $P_{F,t}$, with import share $\alpha \in (0,1)$.

Optimality Conditions:

- Zero bond condition:

$$D_t = 0, \tag{2}$$

- Labor supply:

$$\frac{W_t}{P_t} = N_t^{\phi} C_t, \tag{3}$$

- Euler equation:

$$\beta E_t \left[\frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} (1+I_t) \right] = 1, \tag{4}$$

- Domestic goods demand:

$$C_{H,t} = (1-\alpha) \left(\frac{P_t}{P_{H,t}} \right) C_t, \tag{5}$$

- Foreign goods demand:

$$C_{F,t} = \alpha \left(\frac{P_t}{P_{F,t}} \right) C_t, \tag{6}$$

- CPI definition:

$$P_t = P_{H,t}^{1-\alpha} P_{F,t}^{\alpha}, \tag{7}$$

D.1.2 Domestic Goods Firms

Problem Solved: Monopolistically competitive firms produce $Y_{H,t}$ using labor N_t, setting prices with Rotemberg adjustment costs:

$$\max_{P_{H,t}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{C_t}{C_{t+\tau}} \frac{P_t}{P_{t+\tau}} \left[P_{H,t}(i) Y_{H,t+\tau}(i) - \frac{W_{t+\tau}}{(1+\zeta_t) a_h} Y_{H,t+\tau}(i) - \frac{\kappa}{2} \left(\frac{P_{H,t+\tau}(i)}{P_{H,t+\tau-1}(i)} - \pi \right)^2 P_{H,t+\tau} Y_{H,t+\tau} \right],$$

subject to demand: $Y_{H,t+\tau}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t+\tau}}\right)^{-\epsilon} Y_{H,t+\tau}$, where $\epsilon > 1$ is the elasticity of substitution, $\zeta_t \geq 0$ is an optional production subsidy (e.g., $\zeta_t = \frac{1}{\epsilon-1}$ or $\zeta_t = 0$), $\kappa > 0$ is the adjustment cost parameter, π is steady-state inflation, and production is $Y_{H,t+\tau} = a_h N_{t+\tau}$, with $a_h > 0$.

Optimality Conditions:

- Subsidy (optional):

$$\zeta_t = \frac{1}{\epsilon - 1}, \tag{0.1}$$

- Price setting:

$$\begin{aligned} &\left[1 - \epsilon + \epsilon \frac{W_t}{(1+\zeta_t) a_h P_{H,t}} - \kappa \frac{P_{H,t}}{P_{H,t-1}} \left(\frac{P_{H,t}}{P_{H,t-1}} - \pi \right)\right] Y_{H,t} \\ &+ \beta \kappa E_t \left[\frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \left(\frac{P_{H,t+1}}{P_{H,t}} \right)^2 \left(\frac{P_{H,t+1}}{P_{H,t}} - \pi \right) Y_{H,t+1} \right] = 0 \end{aligned} \tag{8}$$

- Aggregate output:

$$Y_{H,t} = a_h N_t, \tag{9}$$

D.1.3 Commodity Sector Firms

Problem Solved: Competitive firms produce commodities $Y_{C,t}$ using domestic goods inputs $M_{H,t}$, maximizing profits:

$$\max_{M_{H,t}} P_{C,t} Y_{C,t} - P_{H,t} M_{H,t},$$

subject to: $Y_{C,t} = a_c M_{H,t}^v$, where $v \in (0,1)$, $a_c > 0$, and $P_{C,t} = \mathcal{E}_t P_{C,t}^*$. Commodities are sold exclusively to the foreign economy at price $P_{C,t}$.

Optimality Conditions:

- Input demand:

$$P_{C,t} v a_c M_{H,t}^{v-1} = P_{H,t}, \tag{10}$$

- Production:

$$Y_{C,t} = a_c M_{H,t}^v, \tag{11}$$

D.1.4 Monetary Authority

Problem Solved: Sets the nominal interest rate I_t via a Taylor rule.

Optimality Conditions:

- Taylor rule:

$$1 + I_t = (1 + I_{stst})^{1-\rho_i}(1 + I_{t-1})^{\rho_i} \left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi(1-\rho_i)}, \tag{12}$$

D.1.5 Foreign Economy

Role: Sets the stationary world commodity price $P^*_{C,t}$ and constant foreign goods price $P^*_{F,t} = p_{f,stst} = 1$.

Equation:

- Commodity price process:

$$\log P^*_{C,t} = \rho_c \log P^*_{C,t-1} + \sigma_c \varepsilon_{C,t}, \quad \varepsilon_{C,t} \sim N(0, 1), \tag{13}$$

D.1.6 Market Clearing and Additional Conditions

Market Clearing:

- Domestic goods market:

$$Y_{H,t} = C_{H,t} + M_{H,t}, \tag{14}$$

- Law of one price:

$$P_{F,t} = \mathcal{E}_t p_{f,stst}, \tag{15}$$

Additional Conditions:

- CPI inflation:

$$\pi_t = \frac{P_t}{P_{t-1}}, \tag{16}$$

- Commodity price definition:

$$P_{C,t} = \mathcal{E}_t P^*_{C,t}, \tag{17}$$

- Firm profits:

$$\Psi_t = P_{H,t}Y_{H,t} - W_t N_t + P_{C,t}Y_{C,t} - P_{H,t}M_{H,t}, \tag{18}$$

D.2 Stationary Variables Definitions

Nominal variables are divided by the CPI P_t, which grows at the steady-state inflation rate π, to eliminate non-stationarity. The world commodity price $P^*_{C,t}$ is stationary. Stationary variables use lowercase notation:

- Consumption: $c_t = C_t$.
- Labor: $n_t = N_t$.
- Real bond holdings: $d_t = \frac{D_t}{P_t}$.
- Domestic goods consumption: $c_{h,t} = C_{H,t}$.
- Foreign goods consumption: $c_{f,t} = C_{F,t}$.
- Domestic output: $y_{h,t} = Y_{H,t}$.
- Commodity input: $m_{h,t} = M_{H,t}$.
- Commodity output: $y_{c,t} = Y_{C,t}$.
- Interest rate: $i_t = I_t$.
- Inflation: $\pi_t = \frac{P_t}{P_{t-1}}$.
- Real wage: $w_t = \frac{W_t}{P_t}$.
- Relative domestic price: $p_{h,t} = \frac{P_{H,t}}{P_t}$.
- Relative foreign goods price: $p_{f,t} = \frac{P_{F,t}}{P_t}$.
- Relative commodity price: $p_{c,t} = \frac{P_{C,t}}{P_t}$.
- Real firm profits: $\psi_t = \frac{\Psi_t}{P_t}$.
- Real exchange rate: $e_t = \frac{\mathcal{E}_t p_{f,stst}}{P_t}$, where $p_{f,stst} = 1$.
- Relative commodity price: $p^*_{c,t} = P^*_{C,t}$ (stationary).
- Subsidy: $\zeta_t = \frac{1}{\epsilon - 1}$ (optional).

D.3 Stationarized Equations

The 18 stationarized equations, with lowercase variables, are:

0.1 Subsidy (optional):

$$\zeta_t = \frac{1}{\epsilon - 1}, \tag{0.1}$$

0.2 Steady-state interest rate:

$$i_{stst} = \frac{1}{\beta} - 1, \tag{0.2}$$

1 Household budget constraint:

$$c_t + \frac{d_t}{1 + i_t} = w_t n_t + \frac{d_{t-1}}{\pi_t} + \psi_t, \tag{1}$$

2 Zero bond condition:

$$d_t = 0, \tag{2}$$

3 Labor supply:

$$w_t = n_t^{\phi} c_t, \tag{3}$$

4 Euler equation:

$$\beta E_t \left[\frac{c_t}{c_{t+1}} \frac{1}{\pi_{t+1}} (1 + i_t) \right] = 1, \tag{4}$$

5 Domestic goods demand:

$$c_{h,t} = (1 - \alpha) \left(\frac{1}{p_{h,t}} \right) c_t, \tag{5}$$

6 Foreign goods demand:

$$c_{f,t} = \alpha \left(\frac{1}{p_{f,t}} \right) c_t, \tag{6}$$

7 CPI definition:

$$1 = p_{h,t}^{1-\alpha} p_{f,t}^{\alpha}, \tag{7}$$

8 Price setting (Rotemberg, symmetric equilibrium):

$$\left[1-\epsilon+\epsilon\frac{w_t}{(1+\zeta_t)a_h p_{h,t}}-\kappa\frac{p_{h,t}\pi_t}{p_{h,t-1}}\left(\frac{p_{h,t}\pi_t}{p_{h,t-1}}-\pi\right)\right]y_{h,t} \\ +\beta\kappa E_t\left[\frac{c_t}{c_{t+1}}\frac{1}{\pi_{t+1}}\left(\frac{p_{h,t+1}\pi_{t+1}}{p_{h,t}}\right)^2\left(\frac{p_{h,t+1}\pi_{t+1}}{p_{h,t}}-\pi\right)y_{h,t+1}\right]=0 \tag{8}$$

9 Aggregate output:

$$y_{h,t}=a_h n_t, \tag{9}$$

10 Commodity input demand:

$$p_{c,t} v a_c m_{h,t}^{v-1}=p_{h,t}, \tag{10}$$

11 Commodity production:

$$y_{c,t}=a_c m_{h,t}^{v}, \tag{11}$$

12 Taylor rule:

$$1+i_t=(1+i_{stst})^{1-\rho_i}(1+i_{t-1})^{\rho_i}\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi(1-\rho_i)}, \tag{12}$$

13 Commodity price process:

$$\log p^*_{c,t}=\rho_c\log p^*_{c,t-1}+\sigma_c\varepsilon_{c,t}, \quad \varepsilon_{c,t}\sim N(0,1), \tag{13}$$

14 Domestic goods market clearing:

$$y_{h,t}=c_{h,t}+m_{h,t}, \tag{14}$$

15 Law of one price:

$$p_{f,t}=e_t, \tag{15}$$

16 CPI inflation:

$$\pi_t=\frac{P_t}{P_{t-1}}, \tag{16}$$

17 Commodity price definition:

$$p_{c,t}=e_t p^*_{c,t}, \tag{17}$$

18 Firm profits:

$$\psi_t = p_{h,t}y_{h,t} - w_t n_t + p_{c,t}y_{c,t} - p_{h,t}m_{h,t}, \tag{18}$$

D.4 Economic Flows

Households supply labor (N_t) to domestic firms, receiving wages (W_t) and profits (Ψ_t), and consume domestic ($C_{H,t}$) and foreign goods ($C_{F,t}$) with no bond holdings ($D_t = 0$). Domestic firms produce output ($Y_{H,t}$) at price $P_{H,t}$, supplying households and commodity firms ($M_{H,t}$). Commodity firms produce output ($Y_{C,t}$) sold to the foreign economy at price $P_{C,t}$, using domestic inputs ($M_{H,t}$). The monetary authority sets the interest rate (I_t) based on inflation (Π_t). The foreign economy sets $P^*_{C,t}$ and $P^*_{F,t}$, affecting prices via $\mathcal{E}_t$. The balanced current account ($P_{F,t}C_{F,t} = P_{C,t}Y_{C,t}$) is satisfied.

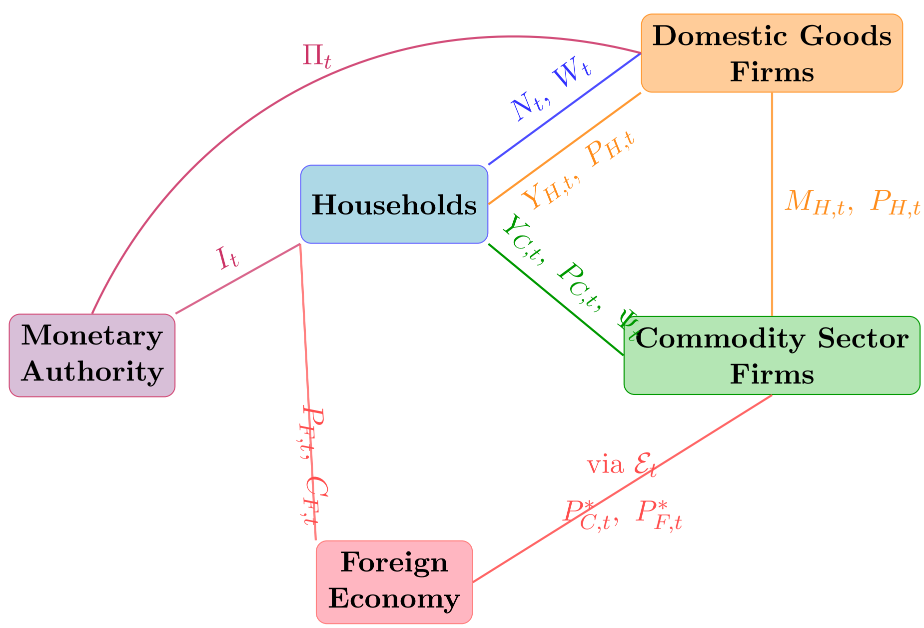

Figure D-1: Flowchart of economic interactions in the simplified DSGE model.

D.5 Model Closure

The nonstationary model has 18 endogenous variables: C_t, N_t, D_t, $C_{H,t}$, $C_{F,t}$, P_t, W_t, $P_{H,t}$, $Y_{H,t}$, $M_{H,t}$, $Y_{C,t}$, I_t, $\mathcal{E}_t$, Π_t, $P_{F,t}$, $P_{C,t}$, $P^*_{C,t}$, Ψ_t; and 18 equations (0.1–0.2, 1–18). The stationary model has 18 endogenous variables: $c_t, n_t, d_t, c_{h,t}, c_{f,t}, w_t, p_{h,t}, y_{h,t}, m_{h,t}, y_{c,t}, i_t, e_t, \pi_t, p_{f,t}, p_{c,t}, \psi_t, p^*_{c,t}, \zeta_t$;

and 18 equations (0.1–0.2, 1–18). The exogenous variables are $\varepsilon_{C,t}$ a *IID* shock and $p_{f,stst} = 1$.

D.6 Steady-State Solution

The steady state is computed by setting all time-subscripted variables to their steady-state values ($x_t = x_{t-1} = x_{t+1}$) and solving sequentially, assuming $n_t = \bar{n} = \frac{1}{3}$. The inverse Frisch elasticity ϕ is determined in steady state. The steady-state equations are:

0.1 Subsidy (optional):

$$\zeta_t = \frac{1}{\epsilon - 1}, \tag{0.1}$$

0.2 Steady-state interest rate:

$$i_{stst} = \frac{1}{\beta} - 1, \tag{0.2}$$

12 Taylor rule:

$$\pi_t = \pi, \tag{12}$$

4 Euler equation:

$$i_t = \frac{\pi_t}{\beta} - 1, \tag{4}$$

2 Zero bond condition:

$$d_t = 0, \tag{2}$$

13 Commodity price process:

$$p^*_{c,t} = 1, \tag{13}$$

9 Aggregate output:

$$y_{h,t} = a_h n_t, \tag{9}$$

1 Household budget constraint (implied by balanced current account):

$$m_{h,t} = \frac{\alpha v}{1 - \alpha + v\alpha} y_{h,t}, \tag{1}$$

10 Commodity input demand:

$$p_{h,t} = \left[p^*_{c,t} v a_c m_{h,t}^{v-1}\right]^{\alpha}, \tag{10}$$

14 Domestic goods market clearing:

$$c_{h,t} = y_{h,t} - m_{h,t}, \tag{14}$$

5 Domestic goods demand:

$$c_t = \frac{1}{1-\alpha} p_{h,t} \left(y_{h,t} - m_{h,t} \right), \tag{5}$$

7 CPI definition:

$$p_{f,t} = \left(p_{h,t} \right)^{\frac{\alpha-1}{\alpha}}, \tag{7}$$

17 Commodity price definition:

$$p_{c,t} = p_{f,t} p^*_{c,t}, \tag{17}$$

11 Commodity production:

$$y_{c,t} = a_c m^v_{h,t}, \tag{11}$$

15 Law of one price:

$$e_t = p_{f,t}, \tag{15}$$

8 Price setting (Rotemberg, symmetric equilibrium):

$$w_t = \frac{\epsilon - 1}{\epsilon} (1 + \zeta_t) a_h p_{h,t}, \tag{8}$$

3 Labor supply:

$$\phi = \frac{\log \left(\frac{w_t}{c_t} \right)}{\log n_t}, \tag{3}$$

6 Foreign goods demand:

$$c_{f,t} = \alpha \left(\frac{1}{p_{f,t}} \right) c_t, \tag{6}$$

18 Firm profits:

$$\psi_t = p_{h,t} y_{h,t} - w_t n_t + p_{c,t} y_{c,t} - p_{h,t} m_{h,t}. \tag{18}$$

Figure D-2 provides the results under both types of monetary policy, i.e., a simple Taylor rule and optimal commitment policy. The results confirm that commodity price shocks are important determinants for inflation and output and hence reinforce our main research objective in the paper for monetary policy to cater for such fluctuations. Optimal commitment policy delivers lower fluctuations in output altogether, as depicted in both the figure and the table D-1 of the results for the standard deviations of the variables.

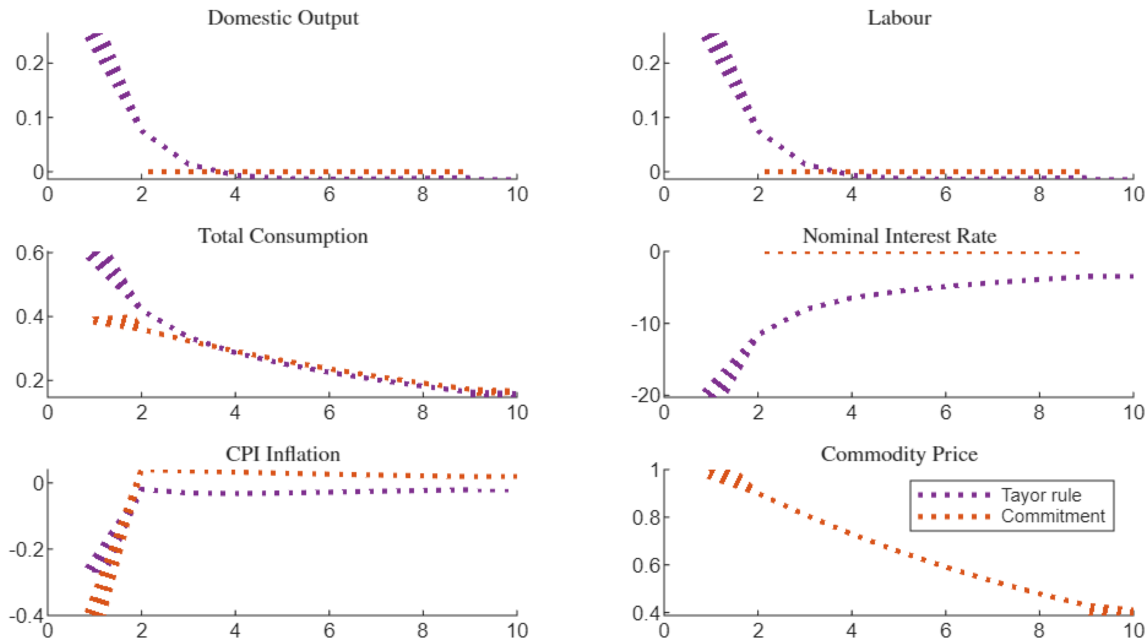

Figure D-2: Optimal Commitment Policy vs Taylor rule

Variables	Taylor Rule	Optimal Commitment
Domestic Output	0.0848	0.0000
Labour	0.0848	0.0000
Total Consumption	0.1408	0.0823
Nominal Interest Rate	5.2582	0.0000
CPI Inflation	0.0776	0.1353
Commodity Price	0.2058	0.2058

Table D-1: Standard deviations of main variables

E Impulse response functions with confidence bands

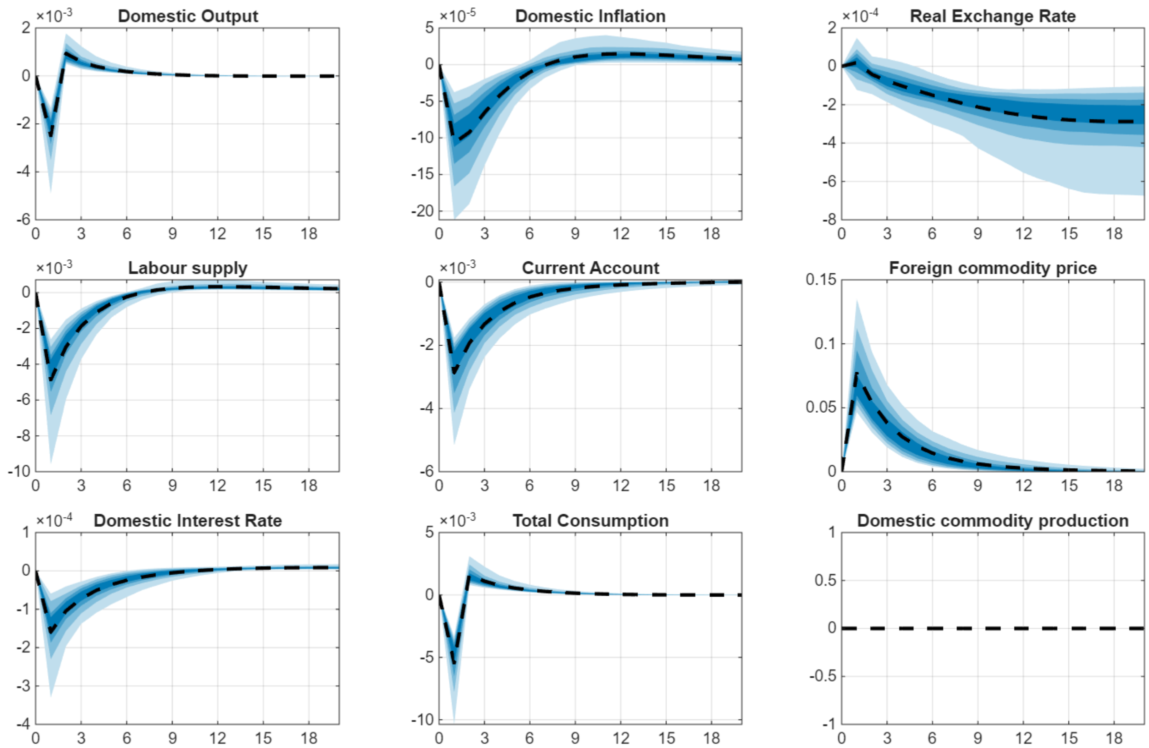

Figure E-3: Generalized IRFs for a commodity price shock

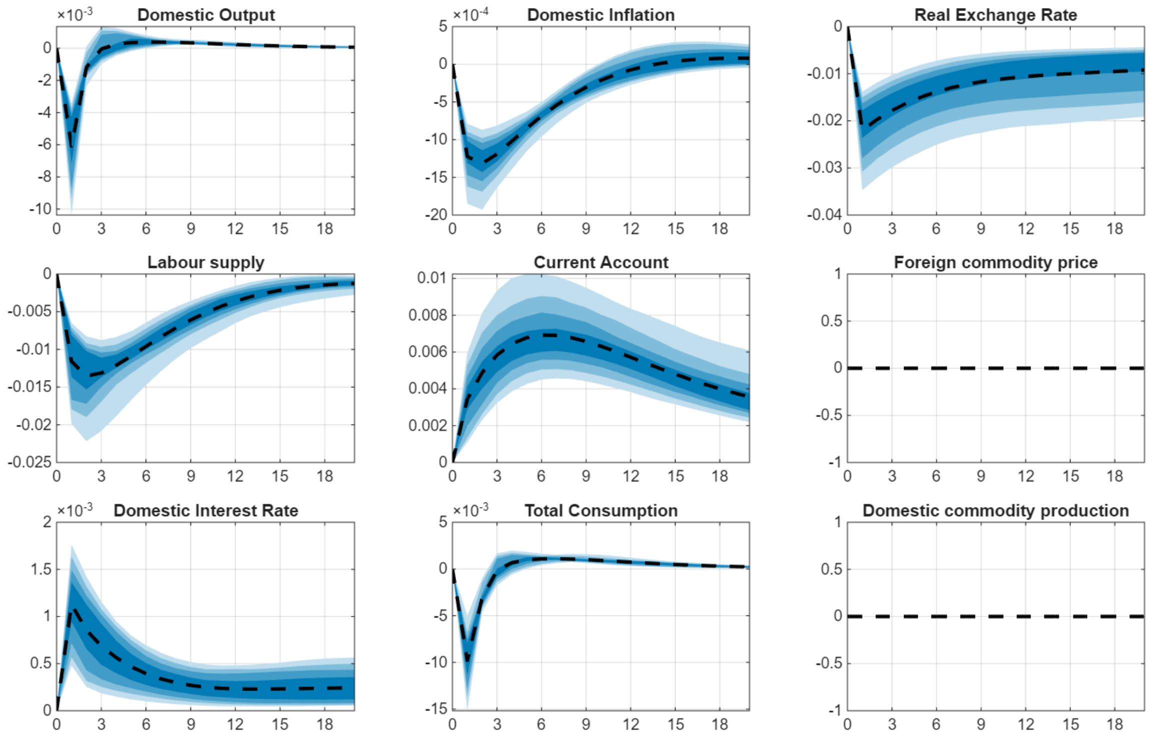

Figure E-4: Generalized IRFs for a monetary policy shock

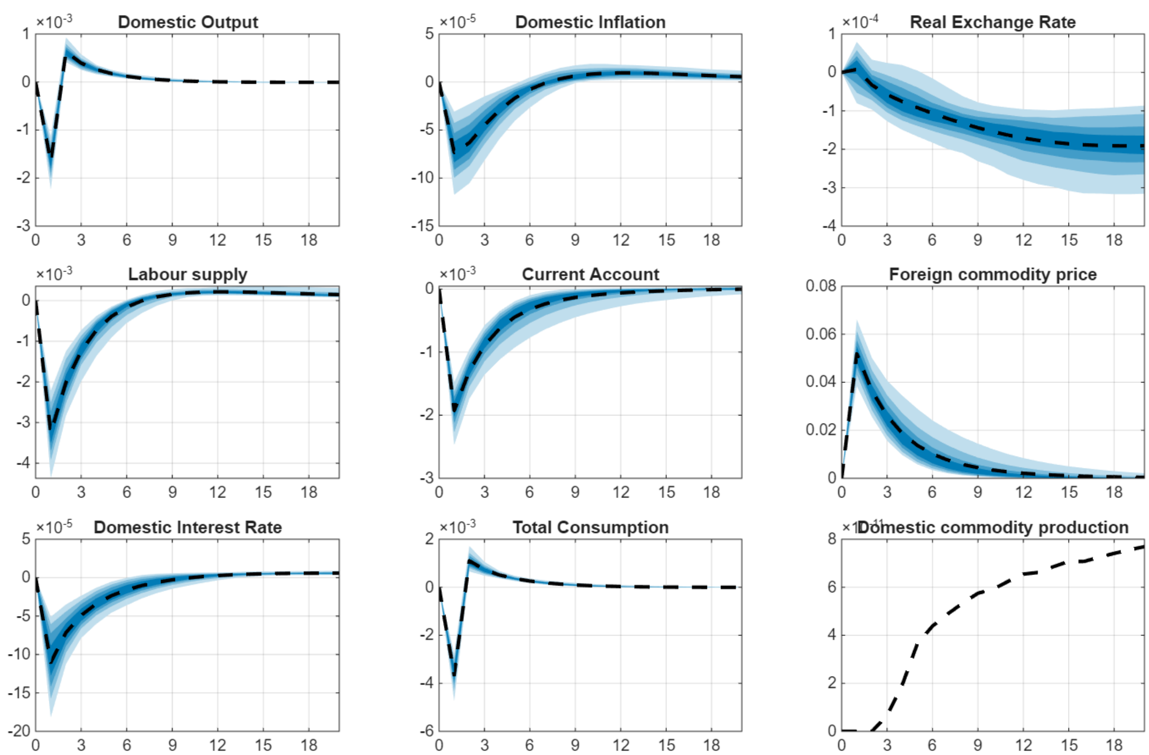

Figure E-5: Regime 1 specific IRFs to a commodity price shock

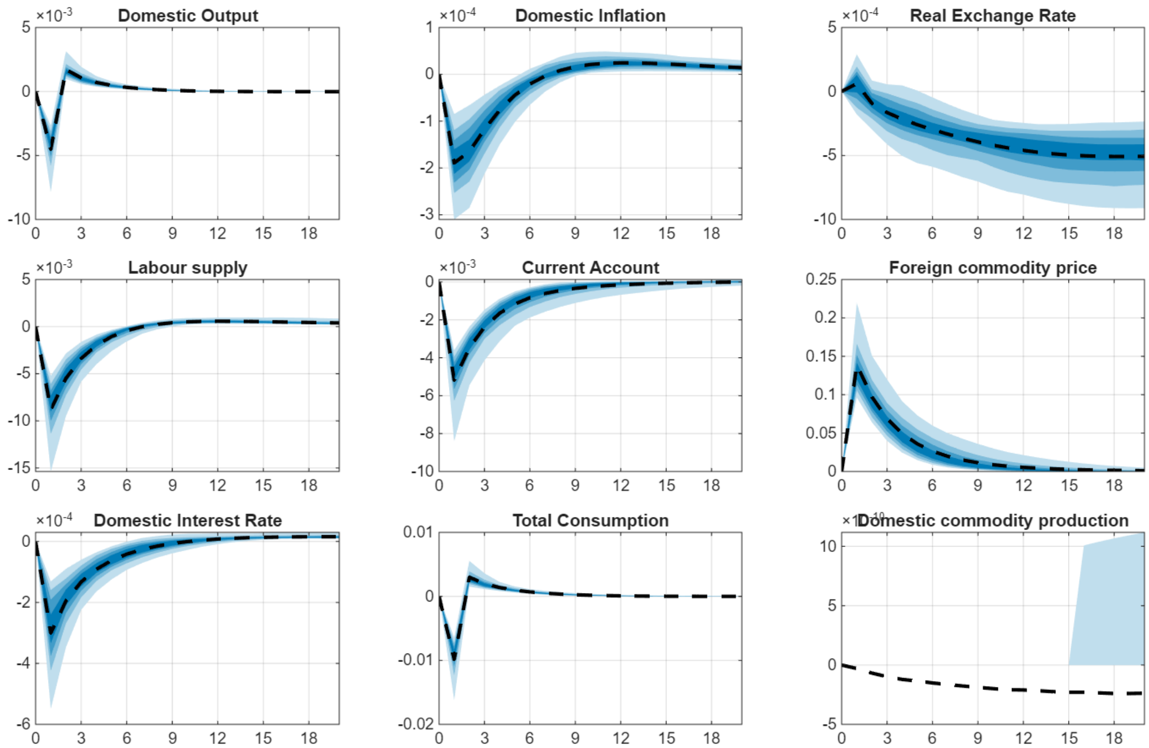

Figure E-6: Regime 2 specific IRFs to a commodity price shock

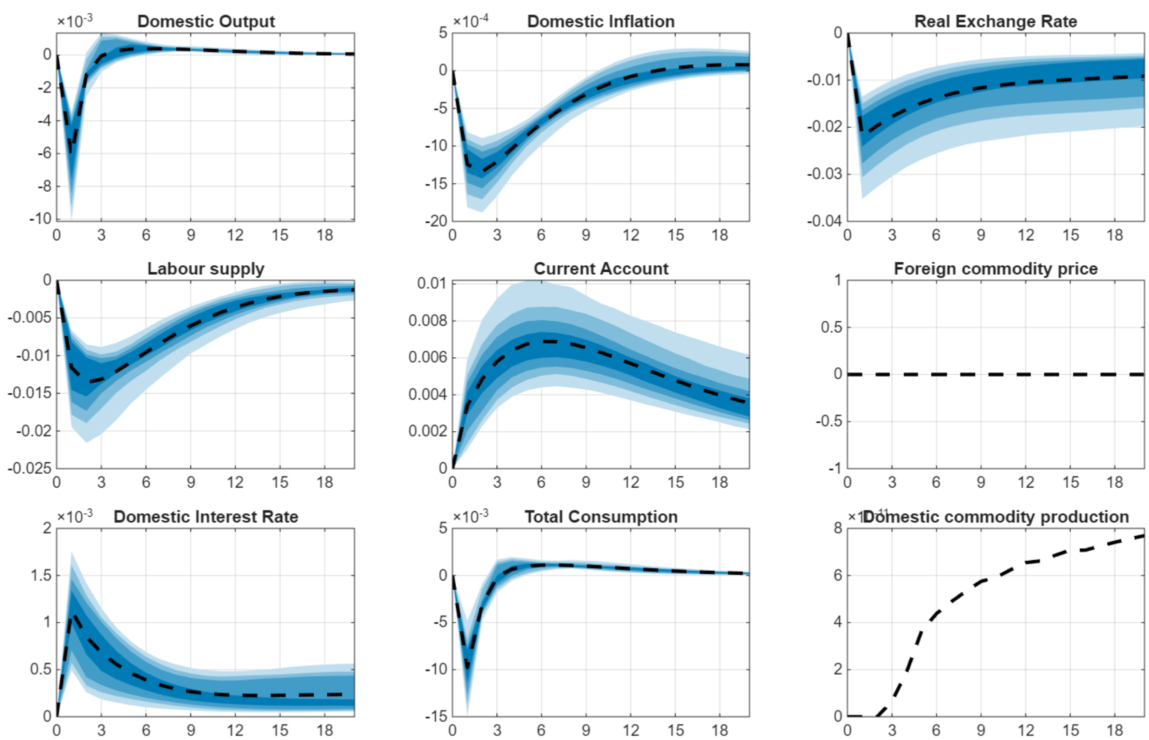

Figure E-7: Regime 1 specific IRFs to a monetary policy shock

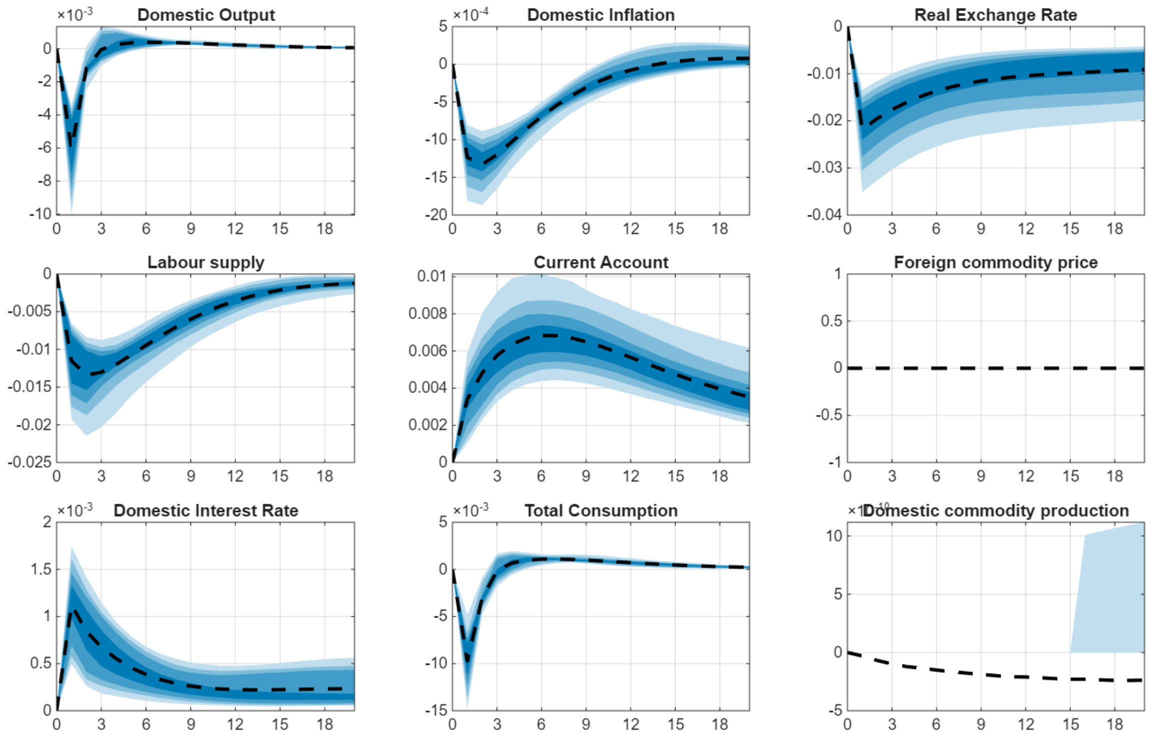

Figure E-8: Regime 2 specific IRFs to a monetary policy shock

F IRFs comparison between constant parameter DSGE and regime switching DSGE

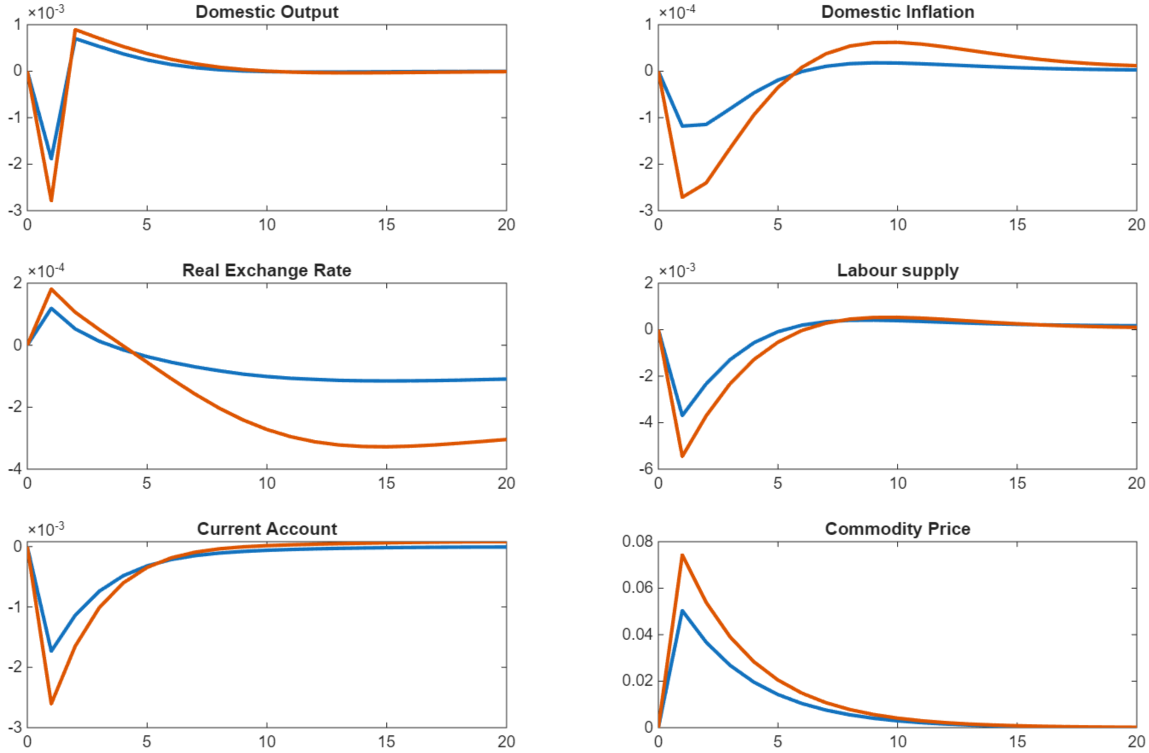

Figure F-9: IRFs comparison for a commodity price shock. In blue the regime switching model and in red the constant parameters model.

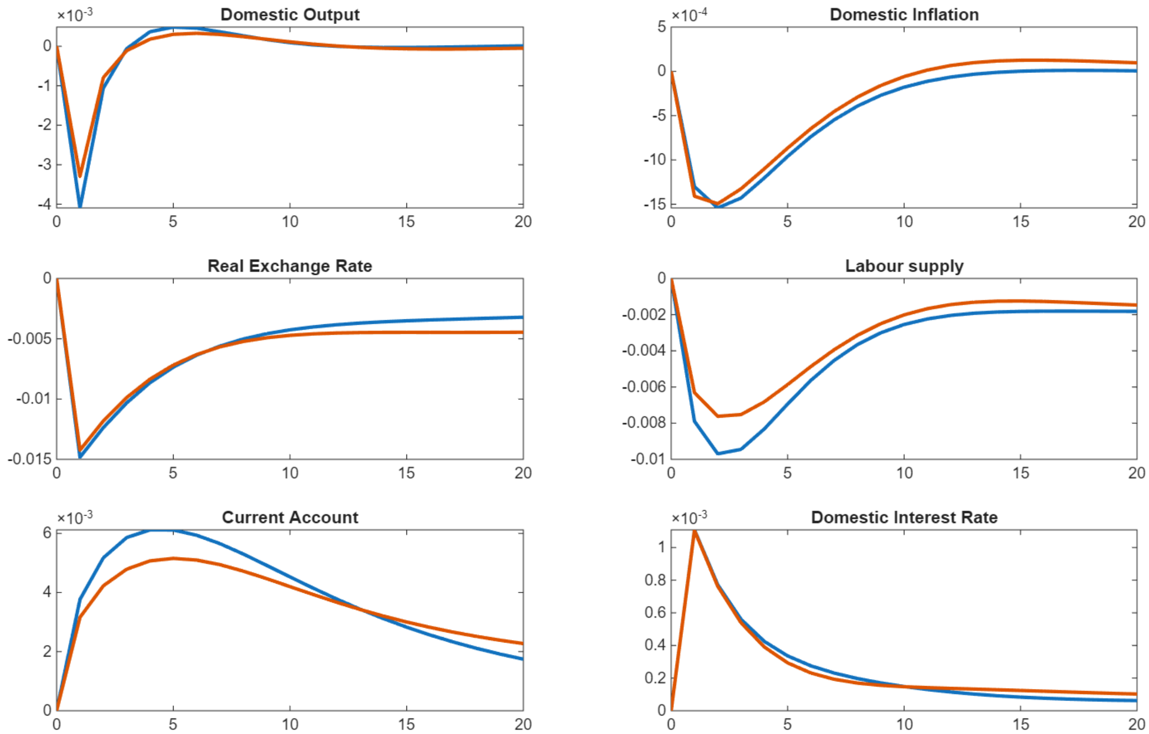

Figure F-10: IRFs comparison for a monetary policy shock. In blue the regime switching model and in red the constant parameters model.

G Variance decomposition of selected variables

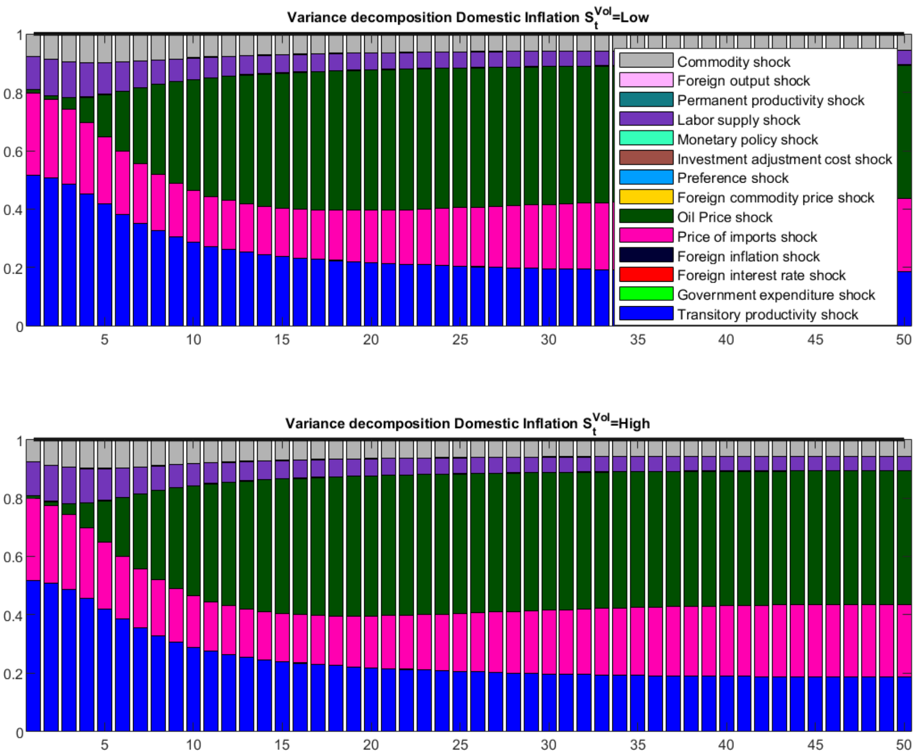

Figure G-11: Variance decomposition Inflation

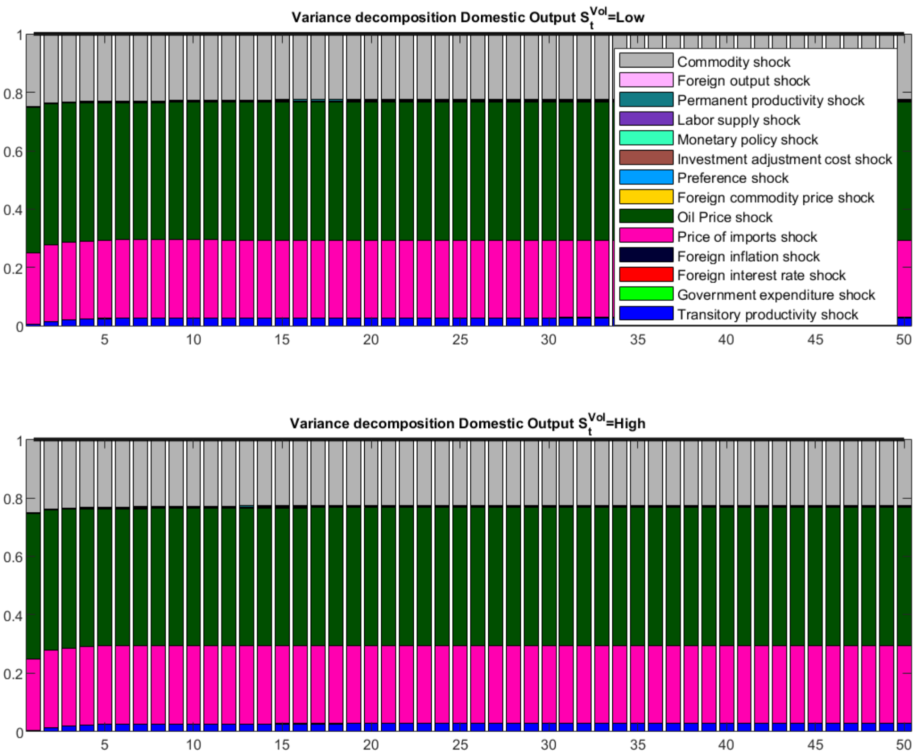

Figure G-12: Variance decomposition domestic output

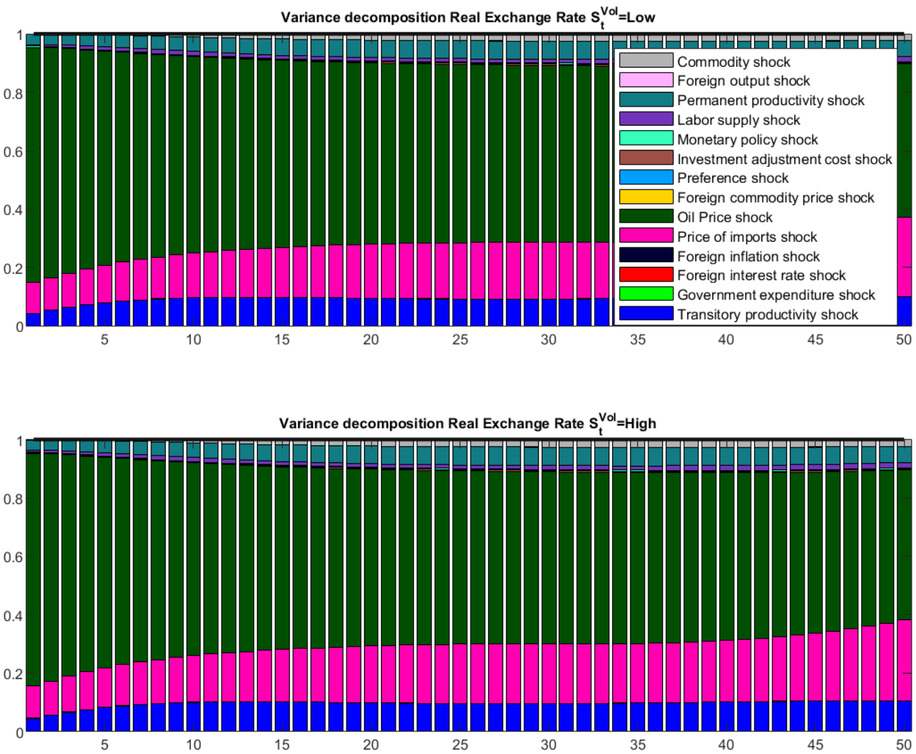

Figure G-13: Variance decomposition real exchange rate

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